vices, $n(\phi)$ dividers/combiners can be used for feeding multi-element antennas where progressive, equal phase delay of the signal is desired. Another interesting mixer application is mentioned in [9].

Acknowledgment

The author thanks F. A. Pelow for his help in building the divider/combiner, and in performing the measurements in the experiment described in Section V.

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On Solving Waveguide Junction Scattering Problems by the Conservation of Complex Power Technique

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Abstract—**Normal mode expansions are used to mode match the tangential electric field at the transverse junction of two cylindrical waveguides. Instead of mode matching the tangential magnetic field the principle of conservation of complex power is invoked and leads, without a matrix inversion, to an expression for the junction's input admittance matrix, as seen from the smaller guide. Simple matrix algebra and the reciprocity theorem then provide the generalized scattering matrix of the two-port (with higher order modes included). It is also shown that the solution satisfies the continuity condition for tangential magnetic field in the junction's aperture. Numerical results are given for parallel plate waveguides with TEM, TE_1, and TM_1 incident fields, numerical convergence being achieved with about ten modes in the smaller waveguide.**

I. Introduction

THEORETICAL and experimental studies of electromagnetic scattering at waveguide junctions have occupied the attention of numerous researchers for several decades. The *variational method* has provided quite accurate dominant mode solutions to a wide variety of waveguide discontinuity problems [1], [2]. Since the advent of fast digital computers interest has shifted to *numerically oriented techniques* due to their broad scope. However, among other restrictions [3]–[6], their range of application is generally limited to two-dimensional problems. A rather complete listing of the related literature is given in the dissertation [7] of one of the authors of this paper.

Following a circuit theory approach, Sharp [8] has presented an exact solution for the admittance matrix of a T-junction of rectangular waveguides. The size of this matrix, which includes propagating and evanescent modes of all ports, makes this approach inefficient when the scattering matrix of the junction or of a single port is of interest. Wexler [9], in dealing with the transverse junction of cylindrical waveguides, begins with continuity equations of transverse $\boldsymbol{E}$ and $\boldsymbol{H}$, the field being expressed in terms of waveguide modes, and obtains a solution for the mode coefficients. Though the generalized form of reciprocity in n-port junctions, as derived in [7], could be employed to solve the reverse problem, i.e., when the incidence direction

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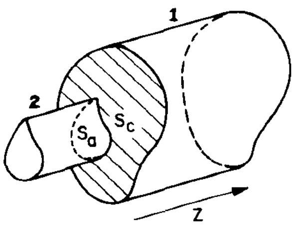

Fig. 1. Junction of two uniform cylindrical waveguides.

is reversed, an approach similar in nature to that used for the forward problem was suggested; this method suffers from relative convergence [4] difficulties.

The *moment method* with mixed basis functions has been applied to the region inside a parallel plate waveguide ([10], [11]). While best suited for two-dimensional problems due to the matrix size involved, the method is also liable to suffer from the relative convergence phenomenon [4]. Though otherwise generating satisfactory results, the *geometrical theory of diffraction* ([12], [13]) fails to perform in cutoff waveguides. The ray diagram is complicated and accuracy is questionable unless the linear dimensions of the scatterer are reasonably large [6].

In the steady state and for the lossless region with vanishingly small thickness at the transverse junction between the two waveguides the net complex power flowing into the region is zero. This power may be expressed in terms of the tangential electric fields on either side of the junction. In the past, this principle of conservation of complex power has been used to determine the admittance of open-ended waveguides [14] and of a dipole antenna [15]. However, in the latter two cases the aperture field and dipole current were assumed to be known *a priori*. Diamond [16] has used the principle with infinite planar waveguide arrays. MacPhie and Zaghloul [17] focused on a single open-ended rectangular waveguide with infinite conducting flange and using the correlation functions of the tangential electric fields in the aperture together with the conservation of complex power principle deduced both the terminal admittance matrix and the radiation pattern of the flanged waveguide antenna.

The problem at hand is that of a closed system, two cylindrical waveguides with a transverse planar junction (see Fig. 1) and with an arbitrary field incident from the smaller guide (guide 2). In Section II, we expand the fields in each guide in terms of the TE and TM normal modes with each mode's amplitude conveniently represented by the element of a column vector. Mode matching the tangential electric field at the junction yields a matrix equation for the mode amplitude vector for the larger guide in terms of the mode amplitude vector for the smaller. The principle of complex power conservation provides a second equation involving Hermitian forms for the incident and transmitted waveguide powers. Combining the two equations results in the elimination of the mode amplitude vector of the larger guide and an expression for the junction's input admittance matrix (as seen from the smaller guide). The associated scattering matrix, involving incident and scattered mode amplitude vectors in the smaller guide, is then determined by matrix manipulation.

The reciprocity theorem and simple matrix algebra permits us to determine the cross scattering matrices involving the mode amplitudes scattered into the larger from the smaller and into the smaller from the larger guide as well as the matrix involving mode amplitudes back scattered into the larger guide due to modes incident from the same waveguide.

In Section III, it is shown that by matching the tangential electric fields at the junction and using the complex power conservation principle, the traditional condition of continuity of the tangential magnetic field is also satisfied.

Section IV treats the case of parallel plate waveguides and presents numerical results which demonstrate convergence of the technique, good convergence being achieved when about 10 modes are used in the smaller waveguide. Moreover, it is to be noted that even with a truncated set of waveguide modes the solution for the propagating modes scattered at the junction *exactly* satisfies the conservation law, i.e., the real power scattered from the junction is equal to the real power incident on the junction.

II. Conservation of Complex Power Technique

In this section, a formal solution to the problem of scattering at the junction of two cylindrical waveguides (Fig. 1) will be presented. The junction's scattering matrix is divided into four submatrices

$$\boldsymbol{S}=\begin{bmatrix} \boldsymbol{S}_{11} & \boldsymbol{S}_{12} \\ \boldsymbol{S}_{21} & \boldsymbol{S}_{22} \end{bmatrix}.$$

The (m,n) element of $\boldsymbol{S}_{ij}$ $(i,j=1,2)$ is the amplitude of the mth mode of guide i due to the unit amplitude nth mode of guide j.

While $\boldsymbol{S}_{22}$ (2 refers to the smaller guide) is computed by the conservation of complex power technique and requires inversion of a matrix of the *same* size, $\boldsymbol{S}_{12}$, $\boldsymbol{S}_{21}$, and $\boldsymbol{S}_{11}$ will be derived successively through matrix equations involving *no* matrix inversion.

Consider the incidence to be from the smaller guide. Let S_a be the aperture surface, S_c be the conductor wall shown by the shaded area, and $S=S_a+S_c$ be the junction surface. Moreover, let the sequences $\{\boldsymbol{E}_{i,n}, \boldsymbol{H}_{i,n}\}$ $(i=1,2)$, where $\boldsymbol{E}_{i,n}$ and $\boldsymbol{H}_{i,n}$ are corresponding electric and magnetic fields, form complete sets of orthogonal modes and be used as basis functions for field expansion in the two guides.

The boundary condition on the electric field across the aperture $(z=0)$ is

$$\begin{aligned} &\sum_n c_{1,n}\boldsymbol{e}_{1,n}=\sum_n (c_{2,n}^{+}+c_{2,n}^{-})\boldsymbol{e}_{2,n}, \qquad \text{over } S_a \\ &\sum_n c_{1,n}\boldsymbol{e}_{1,n}=0, \qquad \text{over } S_c \end{aligned} \tag{1}$$

where $\boldsymbol{e}_{i,n}$ is the transverse component of $\boldsymbol{E}_{i,n}$ at $z=0$ and for $i=1,2$. In the smaller guide $c_{2,n}^{+}$ and $c_{2,n}^{-}$ are, respectively, the amplitudes of the incident and reflected nth mode fields; $c_{1,n}$ is the corresponding amplitude of the transmitted field in the larger guide.

If we scalar multiply (1) by $e^*_{1,m}$ and integrate over S_a+S_c, the orthogonality of the normal modes in the larger waveguide can be used to obtain the equation

$$c_{1,m}=\sum_n H_{mn}c_{2,n} \tag{2}$$

where

$$H_{mn}=Y^*_{01,m}\frac{\int_{S_a} e^*_{1,m}\cdot e_{2,n}da}{p_{1,m}} \tag{3}$$

$Y_{0i,m}$ is the characteristic admittance of the mth mode in guide i ($i=1,2$), and $p_{i,m}$ is the complex power carried by the same mode of unit amplitude. In (2) $c_{2,n}=c^+_{2,n}+c^-_{2,n}$ is the total amplitude coefficient of the nth mode in guide 2.

In matrix notation we can rewrite (2) as

$$\boldsymbol{c}_1=\boldsymbol{H}\boldsymbol{c}_2=\boldsymbol{H}(\boldsymbol{c}^+_2+\boldsymbol{c}^-_2) \tag{4}$$

which we can call the *E-field mode matching equation.*

In the larger guide, just beyond the junction (at $z=0_+$), the modal expansion and a Poynting vector integration may be used to show that the complex power transmitted across the junction is given by the Hermitian form

$$P=\boldsymbol{c}^\dagger_1\boldsymbol{P}_1\boldsymbol{c}_1 \tag{5}$$

where † indicates Hermitian transpose and $\boldsymbol{P}_1$ is a diagonal matrix (due to mode orthogonality) whose mth diagonal element is $p_{1,m}$.

Looking from the smaller guide, the junction may be viewed as a network with N-ports each corresponding to a mode in this guide. The net flow of complex power into this network is

$$P=\frac{1}{2}\boldsymbol{I}^\dagger_2\boldsymbol{V}_2.$$

The nth components of the column vectors $\boldsymbol{I}_2$ and $\boldsymbol{V}_2$ are the nth port's current and voltage, respectively. In terms of the network input admittance matrix $\boldsymbol{Y}_2$ we may write

$$\boldsymbol{I}_2=\boldsymbol{Y}_2\boldsymbol{V}_2$$

or

$$P=\frac{1}{2}\boldsymbol{V}^\dagger_2\boldsymbol{Y}^\dagger_2\boldsymbol{V}_2 \tag{6}$$

where P is the complex power at $z=0$; $\boldsymbol{Y}_i$ ($i=1,2$) is the input admittance matrix of the junction seen by guide i and is a *nondiagonal* square matrix unless the ports are uncoupled; $\boldsymbol{V}_i$ ($i=1,2$) is the equivalent voltage vector and is related to $\boldsymbol{c}_i$ by

$$\boldsymbol{V}_i=\boldsymbol{T}_i\boldsymbol{c}_i, \qquad i=1,2. \tag{7}$$

If the complex power associated with the equivalent voltage and the characteristic admittance of the equivalent transmission line are set equal to the related parameters of the corresponding mode, the mth diagonal element of the diagonal matrix $\boldsymbol{T}_i$ will be

$$T_{i,m}=\sqrt{\frac{2p_{i,m}}{Y^*_{0i,m}}} \tag{8}$$

which may be shown to be real. Substituting (7) into (6), one obtains the Hermitian form for power into the junction

$$P=\frac{1}{2}\boldsymbol{c}^\dagger_2\{\boldsymbol{T}^\dagger_2\boldsymbol{Y}^\dagger_2\boldsymbol{T}_2\}\boldsymbol{c}_2. \tag{9}$$

But it will be recalled that the power transmitted across the junction into the larger guide ((5)) is also given by a Hermitian form. Since the junction is lossless and of vanishingly small volume the principle of conservation of complex power dictates that the two expressions be equal. From (5) and (9) we obtain

$$\boldsymbol{c}^\dagger_1\boldsymbol{P}_1\boldsymbol{c}_1=\frac{1}{2}\boldsymbol{c}^\dagger_2\boldsymbol{T}^\dagger_2\boldsymbol{Y}^\dagger_2\boldsymbol{T}_2\boldsymbol{c}_2. \tag{10}$$

Moreover, $\boldsymbol{c}_1$ and $\boldsymbol{c}_2$ are related by (4), the E-field mode matching equation. Using (4) in (10) gives

$$\boldsymbol{c}^\dagger_2\left\{\boldsymbol{H}^\dagger\boldsymbol{P}_1\boldsymbol{H}-\frac{1}{2}\boldsymbol{T}^\dagger_2\boldsymbol{Y}^\dagger_2\boldsymbol{T}_2\right\}\boldsymbol{c}_2=0.$$

Since the mode amplitude vector is nonzero and arbitrary ($\boldsymbol{c}^+_2$ *is totally* arbitrary), it follows that the input admittance of the junction network, as seen from the smaller waveguide, is

$$\boldsymbol{Y}_2=2\boldsymbol{T}_2^{-1\dagger}\boldsymbol{H}^\dagger\boldsymbol{P}^\dagger_1\boldsymbol{H}\boldsymbol{T}_2^{-1}. \tag{11}$$

Since $\boldsymbol{T}_2$ is a real diagonal matrix its inverse can be easily calculated. As a result *no matrix inversion* is needed to accurately compute the junction input admittance matrix.

The input voltage scattering matrix is

$$\boldsymbol{S}_{v2}=(\boldsymbol{Y}_{02}+\boldsymbol{Y}_2)^{-1}(\boldsymbol{Y}_{02}-\boldsymbol{Y}_2) \tag{12}$$

where $\boldsymbol{Y}_{0i}$ ($i=1,2$) is the characteristic admittance matrix of guide i.

It should be noted that in a variational approach to the problem Collin ([2], p. 322) uses a matrix whose (s,r) element g_{sr} (except for some obvious changes in notation) is given by the corresponding element of $\boldsymbol{Y}_{02}+\boldsymbol{Y}_2$ in (12).

The input scattering matrix is

$$\boldsymbol{S}_{22}=\boldsymbol{T}_2^{-1}\boldsymbol{S}_{v2}\boldsymbol{T}_2. \tag{13}$$

Turning now to $\boldsymbol{S}_{12}$, one can use (4) together with the identity $\boldsymbol{c}^-_2+\boldsymbol{c}^+_2=(\boldsymbol{S}_{22}+\boldsymbol{I})\boldsymbol{c}^+_2$ to show that

$$\boldsymbol{S}_{12}=\boldsymbol{H}(\boldsymbol{S}_{22}+\boldsymbol{I}) \tag{14}$$

where $\boldsymbol{I}$ is the identity matrix.

To determine $\boldsymbol{S}_{21}$ the reciprocity theorem can be used [7] to show that the (i,j) element of $\boldsymbol{S}^T_{12}$ ($\boldsymbol{S}_{12}$ transposed) multiplied by $q_{1,j}/q_{2,i}$, where $q_{m,n}=\iint_{S_m}\boldsymbol{e}_{m,n}\times\boldsymbol{h}_{m,n}\cdot da_m$ ($m=1,2$), is the amplitude of the ith mode of the smaller guide due to the unit amplitude jth mode of the larger guide. Therefore

$$\boldsymbol{S}_{21}=\boldsymbol{Q}_2^{-1}\boldsymbol{S}^T_{12}\boldsymbol{Q}_1 \tag{15}$$

where Q_m $(m=1,2)$ is a diagonal matrix whose ith diagonal element is $q_{m,i}$.

Finally, in view of (4), it may be shown that

$$S_{11}=HS_{21}-I \tag{16}$$

and all four submatrices of the junction scattering matrix S are now known.

III. On Satisfying the Boundary Conditions

We have not satisfied the boundary conditions explicitly. A careful investigation of this matter is needed to justify the technique introduced in this paper. The fields inside the waveguides are represented by the sum of the corresponding waveguide modes. Therefore, the solution automatically satisfies the boundary conditions everywhere but in the aperture plane, $z=0$. Due to mode completeness, (4) ensures that the transverse electric field is continuous over the aperture and vanishes over S_c (in the mean square sense).

Continuity of the transverse magnetic field is represented by

$$\sum_n c_{1,n}Y_{01,n}e_{1,n}=\sum_n (c_{2,n}^{+}-c_{2,n}^{-})Y_{02,n}e_{2,n}, \qquad \text{over } S_a. \tag{17}$$

Scalar multiplying (17) by $e_{2,m}^{*}$ and integrating over S_a gives

$$c_2^{+}-c_2^{-}=Gc_1. \tag{18}$$

The elements of matrix G are

$$G_{mn}=\frac{p_{1,n}^{*}}{p_{2,m}^{*}}H_{nm}^{*}$$

which is equivalent to the following matrix equation:

$$G=P_2^{\dagger -1}H^{\dagger}P_1^{\dagger}. \tag{19}$$

To show that the matrix equation for the continuity of magnetic field (18) is implicitly satisfied by the present technique we can express the law of conservation of complex power as

$$c_1^{\dagger}P_1c_1=(c_2^{+}-c_2^{-})^{\dagger}P_2(c_2^{+}+c_2^{-})=(c_2^{+}-c_2^{-})^{\dagger}P_2c_2 \tag{20}$$

since

$$c_2=c_2^{+}+c_2^{-}.$$

Moreover

$$c_2=(I+S_{22})c_2^{+} \quad \text{or} \quad c_2^{+}=(I+S_{22})^{-1}c_2$$

and

$$c_2^{+}-c_2^{-}=(I-S_{22})c_2^{+}=(I-S_{22})(I+S_{22})^{-1}c_2.$$

Substituting (4) into (20) together with the above relation gives

$$c_2^{\dagger}H^{\dagger}P_1Hc_2=c_2^{\dagger}(I+S_{22}^{\dagger})^{-1}(I-S_{22}^{\dagger})P_2c_2$$

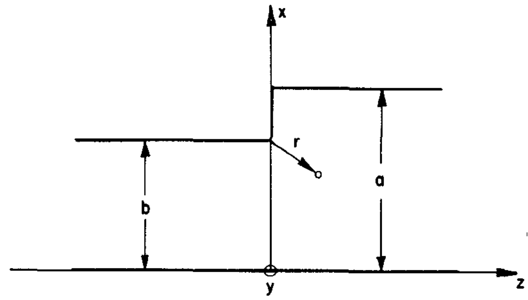

Fig. 2. Junction of two parallel plate waveguides.

which holds for *any incident field*. Thus

$$H^{\dagger}P_1H=(I+S_{22}^{\dagger})^{-1}(I-S_{22}^{\dagger})P_2$$

or

$$I-S_{22}=P_2^{\dagger -1}H^{\dagger}P_1^{\dagger}H(I+S_{22}).$$

Substituting (14) and (19) into the above equation gives

$$I-S_{22}=GS_{12}. \tag{21}$$

Multiplying (21) by c_2^{+} from the right gives

$$c_2^{+}-c_2^{-}=Gc_1$$

which is identical to (18), the equation of continuity for the magnetic field over the aperture surface S_a.

IV. Junction of Parallel Plate Waveguides

Consider two parallel plate waveguides with spacings between plates a and b and their junction at $z=0$, as illustrated in Fig. 2. If the complete sets of orthogonal modes are the TE and TM modes it may easily be verified that the resulting matrix H will be decomposed into two independent matrices for TE and TM excitation.

If the superscripts h and e denote, respectively, the TE and TM modes, then it is straightforward to show that the diagonal matrices Y_{02}, P_1, Q_i, and T_i have diagonal elements given as follows:

$$Y_{02,n}^{h}=\frac{\gamma_{2,n}}{jk_0}Y_0, \qquad Y_{02,n}^{e}=\frac{jk_0}{\gamma_{2,n}}Y_0 \tag{22}$$

$$p_{1,n}^{h}=\frac{a}{4}Y_{01,n}^{h*}, \qquad p_{1,n}^{e}=\frac{a}{2\epsilon_n}Y_{01,n}^{e*} \tag{23}$$

$$q_{i,n}^{h}=p_{i,n}^{h*}, \qquad q_{i,n}^{e}=p_{i,n}^{e*} \tag{24}$$

$$T_{2,n}^{h}=\sqrt{\frac{a}{2}}, \qquad T_{2,n}^{e}=\sqrt{\frac{a}{\epsilon_n}} \tag{25}$$

where

$$\gamma_{1,n}=\sqrt{\left(\frac{n\pi}{a}\right)^2-k_0^2}, \qquad \gamma_{2,n}=\sqrt{\left(\frac{n\pi}{b}\right)^2-k_0^2} \tag{26}$$

$$\epsilon_n=\begin{matrix}1, & n=0\\ 2, & n>0\end{matrix}$$

$$Y_0=\sqrt{\frac{\epsilon_0}{\mu_0}}, \qquad k_0=\omega\sqrt{\epsilon_0\mu_0}. \tag{27}$$

In (27), ϵ_0, μ_0, and ω are, respectively, the permittivity, permeability, and radian frequency.

The elements of the nondiagonal matrices $\boldsymbol{H}^e$ and $\boldsymbol{H}^h$ are

$$H^e_{mn} = (-)^n \frac{2m\pi}{a^2} \frac{\sin\left(\frac{m\pi b}{a}\right)}{\left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2}, \qquad \frac{m}{a} \neq \frac{n}{b}$$

$$= \frac{b}{a}, \qquad \frac{m}{a} = \frac{n}{b}, \quad m, n = 0, 1, 2, \cdots. \tag{28}$$

$$H^h_{mn} = (-1)^n \frac{2n\pi}{ab} \frac{\sin\left(\frac{m\pi b}{a}\right)}{\left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2}, \qquad \frac{m}{a} \neq \frac{n}{b}$$

$$= \frac{b}{a}, \qquad \frac{m}{a} = \frac{n}{b}, \quad m, n = 1, 2, 3, \cdots. \tag{29}$$

The input admittance matrices with incidence from the smaller guide (guide 2) are

$$\boldsymbol{Y}_2^m = 2(\boldsymbol{T}_2^m)^{-1\dagger} \boldsymbol{H}^{m\dagger} \boldsymbol{P}_1^{m\dagger} \boldsymbol{H}^m (\boldsymbol{T}_2^m)^{-1} \tag{30}$$

where $m = h$ and $m = e$ for the TE and TM cases, respectively. Note that since $\boldsymbol{T}_2^m$ is a diagonal matrix $\boldsymbol{Y}_2^m$ can be obtained *without* matrix inversion.

The voltage scattering matrices corresponding to the load admittances in (30) are

$$\boldsymbol{S}_{v2}^m = (\boldsymbol{Y}_{02}^m + \boldsymbol{Y}_2^m)^{-1}(\boldsymbol{Y}_{02}^m - \boldsymbol{Y}_2^m) \tag{31}$$

with $m = h$ (TE) or e (TM).

Using (31) and (13), and (14)–(16) of Section II, the four submatrices $[S_{22}]$, $[S_{12}]$, $[S_{21}]$, and $[S_{11}]$ for the junction's scattering matrix $\boldsymbol{S}$ can be easily deduced.

The three distinct cases of TEM, TM_1, and TE_1 incident waves were considered with the various matrices truncated to include seven modes in the smaller guide and 20 modes in the larger; this causes $\boldsymbol{H}$ to becomes nonsquare, but as is evident from (30), a square admittance matrix (7×7 in this case) is always obtainable.

From the field-theoretic point of view the actual transverse electric fields in the aperture are of prime importance. Fig. 3 gives this information for the first type of excitation (TEM) by using the truncated solution vectors $\boldsymbol{c}_{1T}$ and $\boldsymbol{c}_{2T}$ in (1). This graph indicates that with a finite number of modes the boundary conditions at $z = 0$ are well satisfied. A study of the edge condition ([18], p. 10) reveals that near the edge ($x = b$), $|\boldsymbol{E}_x| \propto 1/r^{1/3}$ where r is the distance measured from the edge. The computed aperture distributions conform approximately to this type of singularity. However, in the larger guide the field ($\boldsymbol{E}_x$) must vanish for $x > b$, giving rise to the sudden drop in the curve of Fig. 3(a) from a maximum to almost zero in the neighborhood of $x = b$.

The magnitude and phase of the reflection coefficient ρ and transmission coefficient τ for different incident modes are plotted in Figs. 4, 5, and 6 as functions of the size of the larger waveguide. As the dimension of the larger waveguide increases so should the mismatch and $|\rho|$; however, the energy being carried away by the new propagating modes tends to decrease $|\rho|$. The two opposing factors generate local minima. This effect is amplified by strong coupling between the incident and new propagating modes with the same transverse distribution.

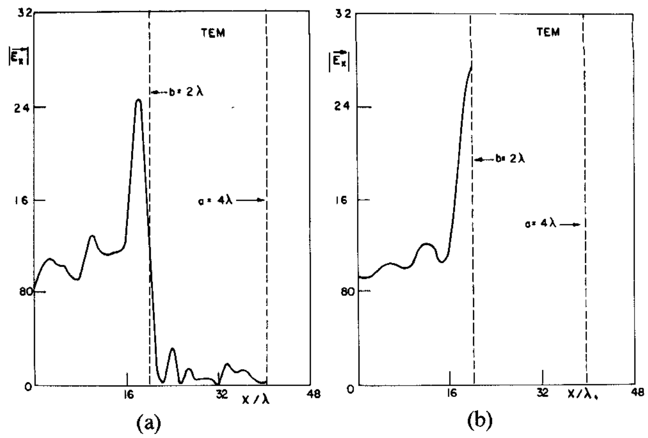

Fig. 3. Distribution of $|E_x|$ along x axis with TEM incidence from smaller waveguide; $a = 0.4\lambda$, $b = 0.2\lambda$. (a) Looking from larger waveguide. (b) Looking from smaller waveguide.

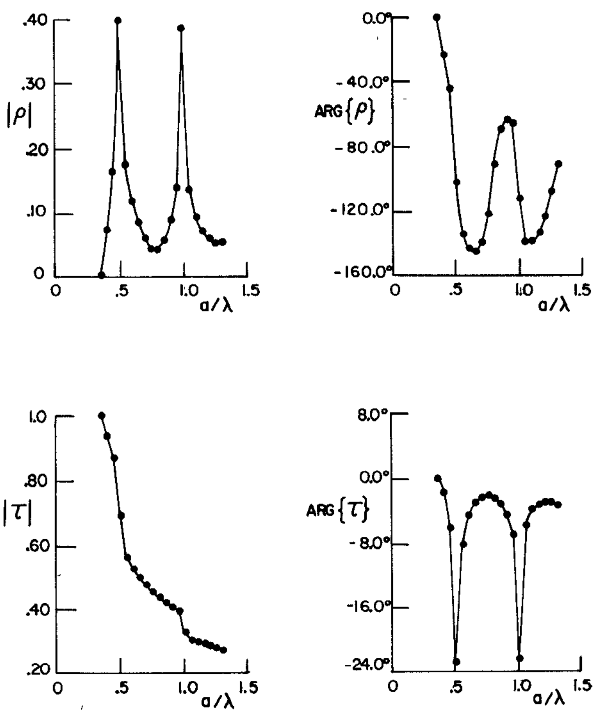

Fig. 4. Reflection coefficient ρ and transmission coefficient τ for TEM incidence as functions of a/λ; incidence from the smaller waveguide; $b = 0.35\lambda$.

In Fig. 6, results from the *Waveguide Handbook* ([1], pp. 298–304) are shown for comparison. These variationally based results are for a limited range ($0.5 < a/\lambda < 1.0$) and at the upper limit the agreement with the present results is not good.

The size of the matrix being inverted in the computer program, i.e., the size of $\boldsymbol{Y}_{02} + \boldsymbol{Y}_2$, is equal to the size of the

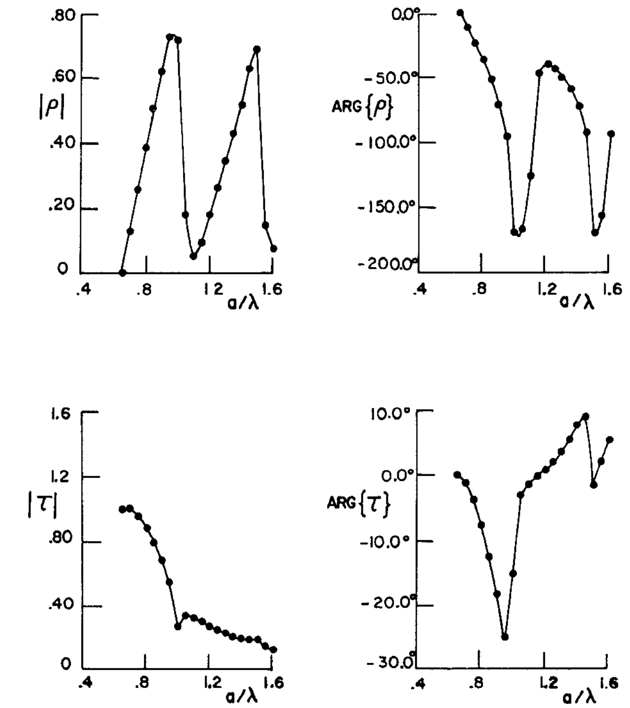

Fig. 5. Reflection coefficient ρ and transmission coefficient τ for TM_1 incidence as functions of a/λ; incidence from smaller waveguide; $b=0.65\lambda$.

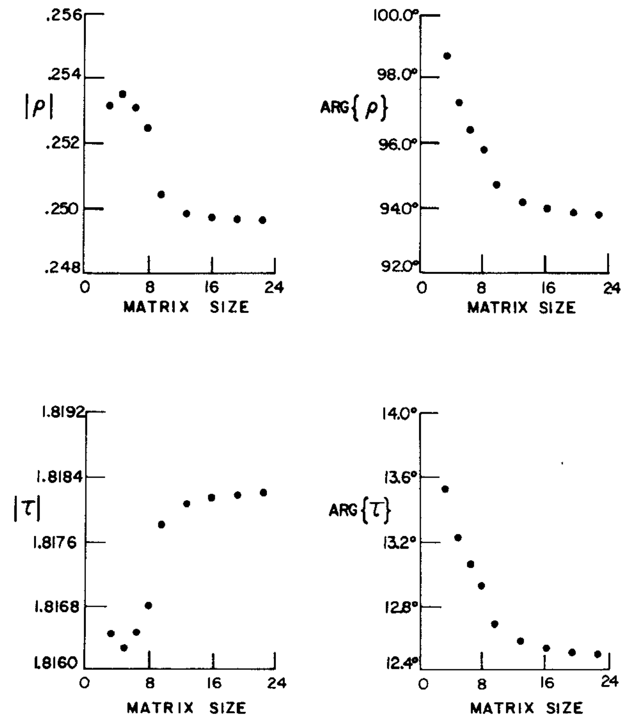

Fig. 7. Convergence of reflection coefficient ρ and transmission coefficient τ as functions of matrix size with TE_1 incidence from larger waveguide; $a=0.95\lambda$, $b=0.55\lambda$.

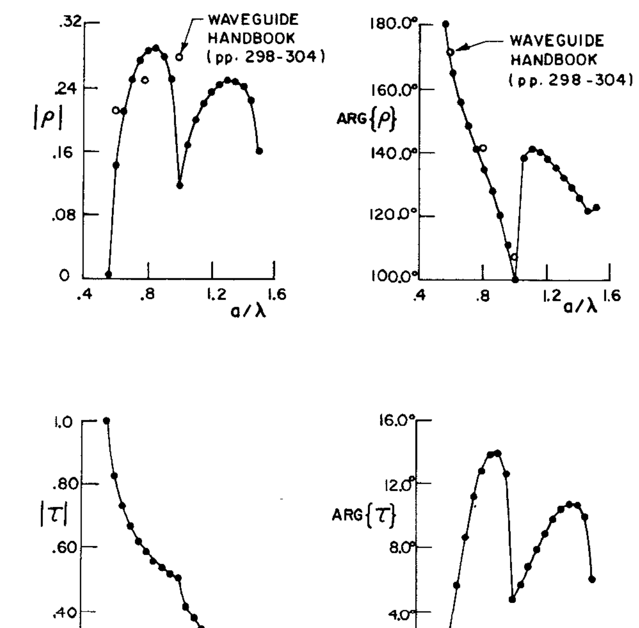

Fig. 6. Reflection coefficient ρ and transmission coefficient τ for TE_1 incidence as functions of a/λ; incidence from the smaller waveguide; $b=0.55\lambda$

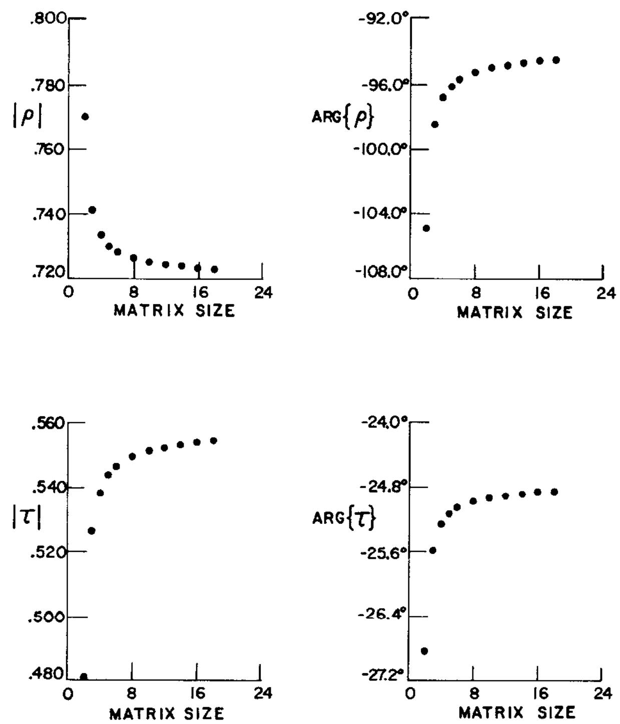

Fig. 8. Convergence of reflection coefficient ρ and transmission coefficient τ as functions of matrix size with TM_1 incidence from smaller waveguide; $a=0.95\lambda$, $b=0.65\lambda$.

scattering matrix S_{22}. To investigate the convergence of the technique, the magnitude and angle of ρ and τ are plotted as functions of the size of S_{22} for both TE and TM cases and incidence from the larger and smaller waveguide in Figs. 7 and 8. The results reveal rapid convergence with matrix sizes of ten or greater, giving accuracy to about three significant figures in most cases. As indicated in Section II, for incidence from the larger waveguide, ρ and τ are derived from the *entire* matrix S_{22}, thereby imposing stronger convergence conditions on S_{22} as a whole.

V. Conclusion

This paper has shown that the conservation of complex power technique, described in Section II, can provide formally exact solutions to certain waveguide junction scattering problems. The method was used to yield time-efficient, numerically convergent solutions for the case of the junction of two parallel plate waveguides, as given in Section IV.

In the near future, a paper on the generalization to the practical problem of scattering at the junction of two *rectangular* waveguides (work that is already complete [7]), will be submitted, together with results on the scattering by a diaphragm with a rectangular iris.

A direct solution to the problem of scattering by a number of semi-infinite septums of arbitrary thicknesses may also be obtained. While a different formulation is needed, the conservation of complex power concept seems applicable to the problem of scattering by a T-junction of parallel plate or rectangular waveguides. In the case of the junction of two waveguides having different dielectric fillings, convergence is expected to be a function of the order of singularity at the edges, as indicated by Mittra and Lee ([18], p. 11).

The above mentioned configurations may be considered as the building blocks for more complex structures such as filters, sidewall couplers, slow wave structures, directional couplers, and finite septums, to state only a few. Applying the generalized scattering matrix concept together with the conservation of complex power technique should provide formally exact, fast and accurate solutions to many of these problems.

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